A Logic of Qualitative Conditional Probability

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Kratzer [10] provides comparative epistemic modals such as "at least as likely (probable) as" with their models in terms of a qualitative ordering on propositions derived from a qualitative ordering on possible worlds. Yalcin [17] shows that Kratzer's model does not validate some intuitively valid inference schemata and validates some intuitively invalid ones. He adopts a model based directly on a probability measure for comparative epistemic modals. His model does not cause this problem. However, as Kratzer [11, p. 25] says, "Our semantic knowledge alone does not give us the precise quantitative notions of probability and desirability that mathematicians and scientists work with", Yalcin's model seems to be unnatural as a model of comparative epistemic modals. Segerberg [14] proposes a complete logic PK of qualitative probability. Gärdenfors [4] simplifies the model of PK. Holliday and Icard [5] prove that not only a probability measure model but also a qualitatively additive measure model and a revised version of Kratzer's model do not cause Yalcin's problem. PK has a binary sentential operator as a qualitative probability operator, whereas Delgrande et al. [1] propose a complete logic of qualitative probability with (m, n)-ary sentential operator with a finite model. We [16] propose a complete logic of qualitative probability without the size of domain. Koopman [6, 7, 8] introduces such a quaternary relation $\precsim$ that $(A, B) \succsim (C, D)$ is interpreted to mean that A given B is at least as probable as C given D. He tries to give sufficient conditions for the existence of such conditional probability measure that

$$(*)(A, B) \succsim (C, D) \text{ iff } P(A, B) \geq P(C, D).$$

However, according to Fine [3, p. 185], Koopman's conditions can only assure that

$$\text{If } (A, B) \succsim (C, D) \text{ then } P(A, B) \geq P(C, D).$$

Koopman's conditions do not assure us of the usual property conditional probability:

$$P(A, B \cap C) = \frac{P(A \cap C, B)}{P(C, B)}$$

Luce [12] gives sufficient conditions for $(*)$ from a measurement-theoretic point of view. Krantz et al. [9] modify Luce's conditions. Domotor [2] gives necessary and sufficient conditions for

(∗) when the sample space is *finite*. Suppes and Zanotti give [15] necessary and sufficient when the sample space has no limitation of its size. Suppes and Zanotti's conditions include **Archimedeanity**. No variant of **Archimedeanity** is expressible not only in the language of propositional logic but also even in that of first-order logic. So we adopt Domotor's conditions as those for the model of our LQCP. Recently Mundici [13] has investigated Koopman's qualitative conditional probability from a logical point of view. However, the complete axiomatization of logic of qualitative conditional probability has been an open problem. The aim of this talk is to propose a new version of complete logic–Logic of Qualitative Conditional Probability (LQCP). （使用言語：日本語）

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