Re-reading Pavel Tichý's The Foundations of Frege's Logic 2

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This is a sequel to our earlier discussion of Pavel Tichý's seminal book *The Foun*dations of *Frege's Logic*. In our last talk, we discussed Tichý's introduction of the fundamental notion of **construction** and two views of mathematics, which differ concerning how they handle the notion of construction. View A treats mathematical formulas as special cases of (incomplete) constructions – calculation schemata – , whereas View B takes mathematical formulas to be purely syntactic entities, which are concerned with mathematical objects. We mainly focused Tichý's initial arguments in favor of View A.

In this talk, we trace one of Tichý's main contentions in the entire book, i.e., the notion of construction is indispensable in interpreting Frege's semantic theory.

Tichý starts his argument by giving textual evidence to show that Frege initially regarded as structured entities Functions (functions in Frege's technical sense of *unsaturated* entities), which were akin to calculation schemata at least in *Begriffsschrift* (as of 1879). However, although it is well-known that in his more mature period Frege explicitly distinguished Functions as unsaturated entities from the courses-of-values of Functions, Tichý claims that Frege's notion of Function became fluctuated (between Function and mapping in the standard set-theoretic or extensional sense) and unstable in Frege's text, due to a potential incoherence among the following three ideas: (i) the unsaturatedness of Functions; (ii) the extensionality thesis (Function as a course-of-value or an extension); (iii) the immediacy thesis (the saturation of Function immediately gives the value).

Tichý further claims that, due to the lack of the notion of construction, Frege was forced to accept the view that an Object (in Frege's technical sense) referred to by a compound expression contains as its parts the references of its syntactic consituents. (E.g., in the case of 100/50, the number 100 and the number 50 are parts of 2.) The view seems to be indeed absurd. Tichý's tries to confirm this point by going into the details of Frege's notational convention, e.g., the issue of how to interpret the parentheses "(", ")" in Frege's Begriffsschrift.

Tichý's diagnosis of these problems is that the problems consist in Frege's having failed to incorporated the notion of construction into his foundational framework. Tichý proposes his own solution to the problems by introducing the notion of construction (and meta-construction). In other words, Tichý essentially recommends Frege to have explicitly adopted View A, although Frege's clear distinction between expressions and entities referred to by the expressions gave a natural inclination towards View B, to which is the almost the entire development of mathematical logic was drawn. In our talk, we will try to critically examine Tichý's diagnosis and solution to the problems and show at least a partial agreement with his judgment concerning the issues, providing some argument for his view focusing on the logical considerations (rather than focusing on Frege scholarship). Time permitting, we will present Tichý's inductive definition of "construction," show how the technical notion of construction can be used to interpret Frege's Begriffsschrift, and discuss some technical issues related to Tichý's technical notion of construction.