On an Information-Theoretic Approach to Better Questions and Answers

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Groenendijk and Stokhof [1] consider the goodness of an answer in terms of the *partition* induced by a *question*. A_Q is defined as the set of all cells of the partition Q that are compatible with an answer A: $A_Q := \{q \in Q : q \cap A \neq \emptyset\}$. In order to compare the goodness of answers, they use a *qualitative ordering* $>_Q$ among answers with relative to Q. Groenendijk and Stokhof propose that when an answer A is *incompatible* with more cells of Q than an answer A', that is, $A_Q \subset A'_Q$, A is considered to be a *better* answer to the question than A'. When $A_Q = A'_Q$, if A does not extra *irrelevant information*, that is, $A' \subset A$, A is considered to be a better answer than A'. Combining both constraints, Groenendijk and Stokhof define A's being a better answer to a question Q than A', in symbols, $A >_Q A'$ as follows: $A >_Q A'$ iff (i) $A_Q \subset A'_Q$, or (ii) $A_Q = A'_Q$ and $A' \subset A$. However, Groenendijk and Stokhof's qualitative approach is *too coarse-grained* to achieve the goal, for we can use this approach only when all cells of a partition are *equiprobable*. In order to overcome the limitations, van Rooij [4] proposes an information-theoretic approach based on *conditional Shannon entropy*. An answer A is better than an answer A' to a question Q iff the *information value* of A is higher than that of A', or in case both of them are the same, the *self-information* of A is lower than that of A': (1) $A >_Q A'$ iff (i) $\mathbf{IV}_Q(A) > \mathbf{IV}_Q(A')$, or (ii) $\mathbf{IV}_Q(A) = \mathbf{IV}_Q(A')$ and $\mathbf{I}(A) < \mathbf{I}(A')$. Moreover, van Rooij considers the goodness of a question. He proposes that a question Q is better than a question Q' with respect to a *mutually exclusive and exhaustive set of hypotheses* H iff the *expected information value* of Q is higher than that of Q', or if both of them are the same, Q is *less fine-grained* than Q': (2) $Q >_H Q'$ iff (i) $\mathbf{EIV}_H(Q) > \mathbf{EIV}_H(Q')$, or (ii) $\mathbf{EIV}_H(Q) = \mathbf{EIV}_H(Q')$ and, for any $q \in Q$, there is such $q' \in Q'$ that $q \subseteq q'$, in symbols, $Q' \subseteq^* Q$. Both $\mathbf{IV}_Q$ and $\mathbf{EIV}_H$ are defined in terms of *conditional Shannon entropy.* $\mathbf{IV}_Q(A)$ is defined as the *reduction of Shannon entropy of a question Q when an answer A is learnt*: $\mathbf{IV}_Q(A) := \mathbf{H}(Q) - \mathbf{H}_A(Q)$, where $\mathbf{H}(Q)$ is the Shannon entropy of Q, and $\mathbf{H}_A(Q)$ is the Shannon entropy of Q conditional on an answer A. So we can rewrite (1) as follows: (3) $A >_Q A'$ iff (i) $\mathbf{H}_A(Q) < \mathbf{H}_{A'}(Q)$, or (ii) $\mathbf{H}_A(Q) = \mathbf{H}_{A'}(Q)$ and $\mathbf{I}(A) < \mathbf{I}(A')$. $\mathbf{EIV}_H(Q)$ is defined as the *average reduction of Shannon entropy of a set of hypotheses H when an answer to a question Q is learnt*: $\mathbf{EIV}_H(Q) := \mathbf{H}(H) - \mathbf{H}_Q(H)$, where $\mathbf{H}_Q(H)$ is the Shannon entropy of H conditional on a question Q. So we can rewrite (2) as follows: (4) $Q >_H Q'$ iff (i) $\mathbf{H}_Q(H) < \mathbf{H}_{Q'}(H)$, or (ii) $\mathbf{H}_Q(H) = \mathbf{H}_{Q'}(H)$ and $Q' \subseteq^* Q$. We [6] provided a *qualitative* approach that is as *fine-grained* as the *information-theoretic (conditional-Shannon-entropy-theoretic)* one, in terms of *measurement theory*, and proposed a new version of complete logic of better questions and answers—Better-Question-and-Answer Logic (BQAL)—the model of which is based on this qualitative approach. Generally, the standard model of economics is based on *global rationality* that requires an *optimising behavior*. But according to Simon [5], cognitive and information-processing constrains on the capabilities of agents, together with the complexity of their environment, render an *optimising behavior* an *unattainable ideal*. Simon dismisses the idea that agents should exhibit global rationality and suggests that they in fact exhibit *bounded rationality* that allows a *satisficing behavior*. It is considered to be not irrational but boundedly rational in questions and answers that we cannot always form a partition induced by a question. How can we deal with this case? This is the *main problem* in this talk. Neither van Rooij [4] nor we [6] consider this case. Rényi [3] provides a clue for solving this problem. He generalizes Shannon entropy to satisfy the weighted quasi-arithmetic mean property. This type of entropy is called Rényi entropy. Moreover, he extends the coverage of Shannon entropy so that it can be applied when we cannot form a partition. We call this type of entropy *incomplete Shannon entropy*. The *aim* of this talk is to furnish a solution to the main problem above by proposing a new version of logic of better questions and answers—Logic-of-Better-Questions-and-Answers (LBQA)—the model of which is based directly on *incomplete conditional Shannon entropy* without depending on measurement theory. BQAL in [6] was a version of *propositional logic*, whereas LBQA in this talk is a version of *first-order logic* the language of which is based on Halpern [2], which can express both *probability* and *incomplete conditional Shannon entropy statements* in an *object language*. （使用言語：日本語）

参考文献

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