Calculation, Proving and Rule

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(This paper is related to joint work with Mathieu Marion on the uniqueness rule.) We discuss rules related to calculation and proving. Although it is often considered that Wittgenstein's view on calculation and rules had been changed by periods (Tractatus, the early transition period, the later transition period, later periods), we propose a different reading of Wittgenstein's texts, according to which his views could be understood coherently through periods, on the base line.
For example, Wittgenstein introduces "numbers" in terms of a version of general operational rule and "calculation" in means of a version of "proof-as-calculation" paradigm, in Tractatus; the "propositions" and the "proofs-calculation" are strictly disconnected; any "proof" cannot be a proof of any "proposition". We demonstrate how these claims of him are kept in the transition periods and later., in principle.

Despite the similarity on the base line, there are, of course, differences in his views on calculation and proofs among the different periods. Wittgenstein "extends" his "Tractatus" view on proofs-calculation in the transition periods; his proofs-as-program view becomes, in the author's opinion, clear in the transition periods (than in the Tractatus period) because of this extended view. His arguments on rule-following calculations (such as inductive-proofs) will be discussed in this context. Such arguments are also related to various issues, such as surveyabiliy, calculability, purity, translatability issues in philosophy of mathematics. We, among others, consider his discussions on commitments to proofs and on accepting proofs and their relation to networking of norm, in the later period, eg. RFM.

At a slight look, the scope of Wittgenstein's discussions on proofs-calculations looks limited to a naïve mathematics: for example, when he talks about arithmetic, he does not say any algebraic equations/properties as sayable (through all periods). However, we note that Wittgenstein suggests further about algebra although algebraic proofs belong to a language game different from the arithmetic one in which we stay with our arithmetic. It is important to think how the arithmetic language game is related to our ordinary life on the one hand, and to the algebra on the other hand. We discuss how Wittgenstein argues on these issues, coherently from some transition period to the later.

Changing the way of calculation-proving would be an aspect-switch, and taking a different way of calculating-proof could commit to a new norm. Wittgenstein's arguments are restricted to the cases of numerical arithmetic rule introductions. We would like to challenge to extend his idea to much wider scope. We remark that the underlying arguments of Wittgenstein's on these are about transformations from a series (progressing sequence) to another series (progressing sequence), or sometime from a sequence of sequences to another sequence of sequences. So, it is not just a commitment to a single new calculation-proving, but to a new (progressive) sequence of calculation-proving (eg. not just 3 x 3 x 3 x 3 to 3^4, but, a progressive sequence to a progressive sequence such as $3^1,3^2,3^3.3^4,3^5,\ldots$). Taking a transformation corresponds to a norm-setting; we discuss how committing to transformations forms a new norm-networking. We discuss what this standpoint would offer to the rule-following controversy.

This standpoint suggests the idea that demonstrating-proving-calculating shows norms and what are being demonstrated, not vice versa; not that norms-propositions provide (propositional) demonstrations-proofs-calculations. We would like to place this idea in some traditional issues of philosophy of mathematics and logic (and further), and try to present some hints for fruitfulness of the idea on the issues.