Wittgenstein's Conception of Mathematical Possibility Revisited

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Why do we accept those mathematical propositions that we do? How can we justify, or at least make it sufficiently intelligible, the fact that we accept them? — One might say this is one of the most central problems (perhaps *the* problem) in the philosophy of mathematics that Wittgenstein tackled throughout. As a preliminary to my talk, I shall clarify here an important part of what I take to be the intent of the problem.

Some might maintain that the problem is in fact illusory since as evidenced by e.g. the opposition between classical mathematicians and intuitionists, there would hardly be any one definite collection of mathematical propositions shared by *every* one of us (or even by *almost all* of us). It is true there is a vast body of mathematical propositions (sentences or formulae) to which virtually everyone finds no difficulty in assenting. But it should be noted, one would say, those seemingly shared propositions are in fact *homonymous*: e.g. classical mathematicians understand them in one way and intuitionists do in quite another — probably incompatible — way. Hence it is illegitimate or at least too optimistic to presuppose from the very start that there could be any plausible collection of mathematical propositions that we (or at least almost all of us) do accept.

Let me articulate the idea underlying this line of argument a bit further. For simplicity, we shall introduce the term the "content" of a (mathematical) proposition and provisionally characterize it as follows: the content of a proposition is just what we should understand of the proposition so that we are entitled to a knower of it (i.e. to a person who understands it). Although this formulation needs to be elaborated much further, the term will serve our present purpose well.

Now, I think the characteristic conception of the content of a mathematical proposition common to those who advocate the line of argument above — i.e. those who are skeptical about starting from the supposition that there is a more or less stable body of *non-homonymous* mathematical propositions shared by (almost) all of us — could be described as follows: they do believe that the content of a mathematical proposition should consist in the way in which the proposition can be *justified*. In other words, one understands a mathematical proposition if and only if she understands the way in which the proposition can be justified (i.e. the justification condition of the proposition).

Let us call this conception of the content of a proposition *the justificatory conception (JC)*. It is true the notion of justification alluded to here also needs to be elaborated still further. (As will be clear from our exposition, the term "justification" is used here in a rather loose sense that permits non-constructive justifications as well). But most would probably agree that we can explain the reason why those who endorse JC are doomed to be skeptical about the supposition of the body of non-homonymous common mathematical propositions as follows.

On the one hand, some of us (e.g. classical mathematicians) find a certain type of justifications (classical proofs) as acceptable and others find other types so while the former refuses the latter types and the latter do the former type: we never accept one and the same means of justifications uniformly. On the other hand, those who endorse JC surely adopt as the criterion of the content of a mathematical proposition the following: the proposition occurring here (i.e. used in one context of our mathematical practice) and the one occurring there (i.e. used in another context) should be counted as identical if and only if one and the same justification condition is associated with both of them. Those who endorse JC are thus inevitably led to the conclusion that the propositions seemingly shared by us are in fact homonymous ones and that the supposition mentioned above is simply false or at least thoroughly without foundations.

Now, the main claim that I should like to make in the talk is the following: it is just freeing us from JC and instead providing with us a different, hitherto not well-appreciated alternative conception of the content of a mathematical proposition that Wittgenstein aimed at throughout. But if so, what is that alternative?

As will be expected, we could and should in the first place make the following remark: according to the alternative conception, the main ingredient of the content of a proposition should be the *use* to which the proposition can be put, or a bit more specifically, the *role* played by it in the process of executing some action by an agent. We would like, however, to put much more emphasis on the *modal* aspect of the problem: the content of a mathematical proposition should be characterized in terms of the (space of) *possibilities* it brings in. I shall expound this aspect of Wittgenstein's conception of the content of a mathematical proposition more fully in the talk.