# Paraconsistent sequential linear-time temporal logic and its application to medical reasoning

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Inconsistency-tolerant temporal reasoning with sequential (i.e., ordered or hierarchical) information is gaining increasing importance in computer science applications such as medical informatics and agent communication. A logical system for representing such reasoning is thus required for obtaining a concrete theoretical basis for such applications. However, to the best of our knowledge, there are no logical systems that can simultaneously represent inconsistency, sequentiality, and temporality. Thus, the aim of our study is to introduce a logical system, both semantically and syntactically, for appropriately representing inconsistency-tolerant temporal reasoning with sequential information.

Hence, we introduce a new logic called *paraconsistent sequential linear-time temporal logic* (PSLTL), which is an extension of the standard *linear-time temporal logic* (LTL). Inconsistency-tolerant reasoning in PSLTL is expressed via a paraconsistent negation connective, and sequential information is represented by sequence modal operators. Temporal reasoning in PSLTL is, of course, expressed by temporal operators used in LTL. We show that a Kripke-style semantics for PSLTL is useful for appropriately handling medical reasoning in a new model checking framework called *paraconsistent model checking*. We also prove some fundamental theorems for PSLTL, such as completeness and cut-elimination theorems, which are obtained via theorems for semantically and syntactically embedding PSLTL into its fragments.

The logic PSLTL proposed is regarded as an extension of both LTL and *Nelson's paraconsistent four-valued logic with strong negation* N4. On the one hand, LTL is known to be one of the most useful temporal logics for verifying concurrent systems. On the other hand, N4 is known to be one of the most important base logics for inconsistency-tolerant reasoning. The combination of LTL and N4 was previously studied in [3], and such a combined logic was called *paraconsistent LTL* (PLTL). Combining LTL with sequence modal operators was also previously studied in [5], and such a combined logic was called *sequence-indexed LTL* (SLTL). PSLTL is then obtained from PLTL by adding sequence modal operators, and is also regarded as a modified paraconsistent extension of SLTL. Thus, PSLTL is a modified extension of both PLTL [3] and SLTL [5].

In the remainder of this abstract, we focus on explanations of an important property of paraconsistent negation and some examples of medical reasoning based on this property. As mentioned, the paraconsistent negation connective $\sim$ used in PSLTL can suitably express inconsistency-tolerant reasoning. One reason why $\sim$ is considered is that it can be added in such a way that the extended logic satisfies the property of *paraconsistency*. A semantic consequence relation $\models$ is called paraconsistent with respect to a negation connective $\sim$ if there are formulas $\alpha$ and $\beta$ such that $\{\alpha, {\sim}\alpha\} \not\models \beta$. In the case of LTL, this implies that there exists a model $M$ and position $i$ of a sequence $\sigma = t_0, t_1, t_2, ...$ of time-points in $M$ with $(M, i) \not\models (\alpha \wedge {\sim}\alpha) \rightarrow \beta$.

It is known that logical systems with paraconsistency can handle inconsistency-tolerant and uncertainty reasoning more appropriately than systems that are non-paraconsistent. We now consider medical reasoning as such reasoning. For example, we do not want $(s(x) \wedge {\sim}s(x)) \rightarrow d(x)$ to be satisfied for any symptom $s$ and disease $d$, where ${\sim}s(x)$ means "person $x$ does not have symptom $s$" and $d(x)$ means "person $x$ suffers from disease $d$," because there may be situations that support the truth of both $s(a)$ and ${\sim}s(a)$ for some individual $a$ but do not support the truth of $d(a)$.

Next, we consider another example. If we cannot determine whether someone is healthy, then the vague concept *healthy* can be represented by asserting the inconsistent formula: $\mathit{healthy}(\mathit{john}) \wedge {\sim}\mathit{healthy}(\mathit{john})$. This is well-formalized in PSLTL because the formula $\mathit{healthy}(\mathit{john}) \wedge {\sim}\mathit{healthy}(\mathit{john}) \rightarrow \mathit{hasCancer}(\mathit{john})$ where $\mathit{hasCancer}(\mathit{john})$ means "John has cancer" is not valid in PSLTL (i.e., PSLTL is inconsistency-tolerant). On the other hand, the formula $\mathit{healthy}(\mathit{john}) \wedge \neg \mathit{healthy}(\mathit{john}) \rightarrow \mathit{hasCancer}(\mathit{john})$ where $\neg$ is the classical negation connective is valid in classical logic (i.e., inconsistency has undesirable consequences). For more information on paraconsistency and its applications, see e.g., [4, 1] and the references therein. It is finally remarked that this talk is based on my papers [1, 2].

## References


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