Dialetheism in philosophy and mathematics

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Dialetheism—the thesis that some contradictions are true—is motivated by apparently sound paradoxical derivations. Beginning from things we seem to know with certainty, such as

- A proposition p is true iff it is the case that p
- An object is in the set of φs iff it is a φ
- An argument is valid iff it necessarily preserves truth
- Some people are bald and some are not

then simple logical steps lead directly to contradictions (the liar, russell's paradox, curry's paradox, and the sorties paradox, respectively). A paradox is an argument with a conclusion that *seems* **false**, but *seems* to follow **validly** from what *seem* like **true** premises. Dialetheism suggests that some paradoxes are *as they seem.*

If dialetheism is correct, it would have vast implications for logic, language, metaphysics, and mathematics. In this talk I will focus mainly on developments in mathematics, in particular on mathematical theories that are based on paraconsistent logic. The *inconsistent mathematics program* is to reconstruct theories from Euclid to modern foundations and beyond, using paraconsistent logic, showing that most mathematical truth/proof does not rely on the assumption of consistency [5, 1, 3, 4]. The program has the interrelated goals of RECAPTURE (reassurance that nothing important is lost) and EXPANSION (where new insights and results are gained). Philosophically, it means having fully expressive theories of truth, sets, validity, and vagueness. Technically, it means studying properties of novel mathematical objects not visible with any other theory.

I will survey past and recent work in dialetheic mathematics, highlighting accomplishments, but also where it falls short [2] and the major challenges it faces. Inconsistent mathematics is not easy, but it is *possible*—and that in itself is an important discovery.[*1]

References

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