Logicism and the Notion of Structure

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Many philosophers these days endorse in one way or another the idea that mathematics is about mathematical structures. Yet, the idea itself is not new. Nor is the notion of arbitrary object, upon which Leon Horsten bases his recent proposal that mathematics be about 'generic structures'.[1] Gottlob Frege once raised objections to the idea that the use of variables in mathematics could be accounted for in terms of such objects.[2] On the other hand, Bertrand Russell once made an attempt to thus explain variables, though he soon abandoned it.[3] Meanwhile, he constantly subscribed to the notion that logic, which he argued was identical to mathematics, must be about a structure—the *logical structure* of the universe, so to speak.

The notion of structure and that of arbitrary object were thus among the topics that these founders of mathematical logic discussed somewhat extensively. It may thus be worthwhile to reconsider these two notions, recalling the possibly old-fashioned yet illuminating discussions of these two philosophers on them, and, at the same time, to examine whether we can consult the current literature on mathematical structuralism to rescue some old ideas these two philosophers once proposed but abandoned due to certain problems.

In view of this, I will undertake two tasks in this paper. One is to examine whether Kit Fine's response to Frege's criticism of arbitrary objects is successful or not,[4] and whether it is available to Horsten, who develops Fine's theory of arbitrary objects into his own view of generic structures. The other is to consider if Fine's theory of arbitrary objects and Horsten's view of generic systems can help to solve the problems which Russell was confronted with when putting forward the notion that logic is about the logical structure of the universe.

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1 Leon Horsten, (forthcoming), 'Generic Structures', *Philosophia Mathematica*.
2 Gottlob Frege, 1979, *Posthumous Writings*, H. Hermes, F. Kambartel and F. Kaulbach eds., Basil Blackwell, Oxford.
3 Bertrand Russell, 1903, *The Principles of Mathematics*, Allen and Unwin: London.
4 Kit Fine, 1983, 'A Defence of Arbitrary Objects', *Proceedings of the Aristotelian Society*, Supplementary Volume 57, pp.55-57.