Mathematical structuralism and arbitrary objects

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According to structuralism in the philosophy of mathematics, mathematics is the science of structure. Mathematical structuralism emerged on the scene as a major position in the philosophy of mathematics in the writings of Dedekind and Hilbert, and became the dominant view in the philosophy of mathematics during the 1980s.

Over the past decades, the discussion among mathematical structuralists has come to revolve about a question of realism. Should we conceive of mathematical structures as platonic entities (non-eliminative structuralism), or are mathematical structures reducible to systems of objects that can be countenanced from a non-platonistic point of view (eliminative structuralism)? In this way, philosophical attention was drawn away from the question what the metaphysical nature of mathematical structure is.

In my talk, I want to move away from the realism debate within structuralism. Instead, I attempt to acquire a deeper insight into the nature of mathematical structures. For this purpose, I seek inspiration in Kit Fine's theory of arbitrary objects. First, I propose that generic systems stand to particular systems in the same way as arbitrary objects stand to particular objects. Second, I propose that mathematical structures should be seen as generic systems. It will be argued that the resulting conception of mathematical structures avoids objections that have been raised against eliminative and against non-eliminative structuralism. In contrast with the notion of structure of eliminative structuralism, the proposed conception of mathematical structure contains mathematical objects. In contrast with the notion on of structure of non-eliminative structuralism, the proposed conception of mathematical structure is not vulnerable to Hellman's permutation argument.